Geomagnetic Effects Associated with the High–Altitude Nuclear Explosion—Supplementary Note—

By

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Abstract

In the previous paper, the writer discussed the geomagnetic effects of the nuclear explosion on 9 th July, 1962 and estimated the enhancement of the effective total conductivity in the ionosphere. In the estimation, the dynamo action in the ionosphere was assumed, ignoring the other geomagnetic effects which may be attributed to any other possible mechanisms. But only the semidiurnal irrotational wind component was taken into consideration. In this paper, the diurnal component of the wind is too considered, together with the semi-diurnal one. Both components of the wind may be the most predominant in the ionosphere, referring to the many analyses of the geomagnetic diurnal variations on the quiet and/or disturbed days and other geophysical data. Thus a revised distribution of the increment of the conductivity near the time of maximum disturbance is obtained. The geomagnetic records of Oct. 22 and 28, 1962 at Kakioka, Memambetsu and Kanoya are reproduced in Appendix II, without analysis.

§ 1. Introduction

In the previous paper, the author preliminarily discussed the dynamo current in the ionosphere, possibly caused by the nuclear explosion on 9th, July, 1962 (1). But only the semi-diurnal wind component Ψ_2^2 was considered. In this paper he treats the problem further to take the diurnal wind into consideration, although the back ground idea is the same as one in the previous paper.

Concerning the increment distribution of the conductivity, many different models of various types may be assumed. For example, the case in which the increment are limited within the zonal region near the geomagnetic or geographic equator and the case where it is predominant along the magnetic line of force through the shot point and others may be possibly considered. But, it is interesting for the understanding the geomangnetic behaviour of the explosion to calculate the dynamo current system with the wind of Ψ_1^1, Ψ_2^2 and the probable distribution of the increment, based on such a usual dynamo theory as one by Chapman, although some other causes of the geomagnetic effects may be considered.

Concerning the wind near the ionosphere, there are many data and reports. Briggs & Spencer reviewed the results by the radio methods (2). Whipple described the photometric results of meteor tails (3). Also, Kellogg & Schilling (4) and Pant (5) deduced the wind, based on the meteorological standpoint. They concluded the wind of 100m/sec in the E layer. While, Taylor (6), Pekeris (7), Wilks (8), Sen and White (9), Chapman and Bartels (10) and Wulf (11) discussed whether the wind contributing to the dynamo theory is of tidal origin or of heating by the sun. Also, the deduced wind system from the Sq dynamo theory by Maeda and/or Kato gives the diurnal wind velocity of 40m/sec in summer and 15m/sec in winter and the semi-diurnal wind velocity of 10m/sec in summer and 20m/sec in winter (12), consistent with the results of Elford and Robertson (13), Briggs and Spencer (2) and Greenhow and Neufeld (14).

On the other hand, Vestine (15), Fukushima (16), Obayashi and Jacobs (17) and Matsushita (18) obtained the favourable wind systems to their dynamo theories of the geomagnetic disturbance field. They assumed the wind contributing to the dynamo action has the velocity potential. The velocity potential has been expanded into a spherical harmonics and practically the first few terms have been treated.

From the geomagnetic data, Ψ_1^1, Ψ_2^2 and Ψ_3^1 are rather moderate, but Maeda (12) obtained Ψ_3^1 is one-fifteenth of Ψ_1^1 and one-tenth of Ψ_2^2. Also Obayashi (17) concluded the wind of $\Psi_2^2+0.2\ \Psi_3^1$ is reasonable for the dynamo theory of Sq and Ds. Thus, the absolute values of Ψ_1^1 and Ψ_2^2 are nearly same and larger than Ψ_3^1. The maximum wind velocities of Ψ_1^1 and Ψ_3^1 occur at the pole, while Ψ_2^2 does in the middle latitude.

And also it is naturally thought that the maximum increment of the conductivity will be near Johnston Island in the case of this disturbance.

Hence Ψ_2^2 may be the leading actor in this disturbance and the examination of only both Ψ_1^1 and Ψ_2^2 ignoring Ψ_3^1 may not result in any important miscalculation. The functional form of each potential here adopted is assumed as follows :

$$\begin{aligned} \Psi_1^1 &= \kappa_1^1 P_1^1 (\cos\theta) \sin(\varphi-\alpha_1^1) \\ \Psi_2^2 &= \kappa_2^2 P_2^2 (\cos\theta) \sin(2\varphi-\alpha_2^2) \end{aligned} \qquad (2)$$

where P_1^1 $(\cos\theta)$ and P_2^2 $(\cos\theta)$ are the Schmidt's functions and (θ, φ) are the geographical colatitude and longitude (reckoned eastwards from the standard meridian through Greenwich).

§ 2. Deduction of the geographical distribution of the increment of the conductivity

We are now going to obtain the most probable distribution of the increment of the conductivity for the wind system of the velocity potential $\Psi_1^1+\Psi_2^2$. That is the most probable expression of the equation (3) in the previous paper. Now $\Delta\sigma$ is assumed as follows,

$$\Delta\sigma=K\ (a_0+a_1\cos\text{Ⓗ}+a_2\cos^2\text{Ⓗ}) \qquad (3)$$

where Ⓗ stands for Ψ in the previous paper. Following the perfectly analogous procedure to the usual dynamo theory of Sq by S. Chapman, the nth term R_n of the spherical harmonic expansion of the current function ofthe dynamo current, with the wind and the conductivity which are expressed by the equation (2) and (3), respectively, may be able to be given as follows ;

$$R_n^t=\kappa_\sigma^\tau CK\sum_{m=-n}^{n} p_n^m P_n^m\ (\cos\theta)\ \sin\ [m(\varphi-\varphi_0)-\alpha] \qquad (4)$$

where p_n^m is a function of a_i's given in Appendix I and $C=-\frac{2}{3}$ gauss. Other notations are usual. Comparing the corresponding terms of the theoretical current function and the equivalent current system for the observed geomagnetic disturbance, p_n^m can be obtained and in turn, a_i's and K can be estimated. The numerals of the coefficients A_n^m and B_n^m in the previous paper were obtained for the associated Legendre Functions and then, they should be corrected by the factor 1 for $m=0$ and $\left[2\frac{(n-m)!}{(n+m)!}\right]^{\frac{1}{2}}$ for $m\neq 0$, in order to be consistent with the coefficients in this paper. The corrected A_n^m's B_n^m's, multiplied by $\frac{2n+1}{n+1}$, are tabulated in Table 1. Assuming $\alpha_1{}'=135°$ and $k_1^1=2.9\times10^{11}$ e. m. u., (19), $\Delta\sigma$ is obtained as follows,

$$\Delta\sigma=4.5\times10^{-8}\ (1+2.6\cos\text{Ⓗ}+25.1\cos^2\text{Ⓗ}),$$

Table 1. The coefficients of spherical harmonic development of the current function

	A_n^m			B_n^m	
n \ m	0	1	2	1	2
1	12.309	−7.185		−24.384	
2	−0.541	−3.907	−5.889	15.145	4.323
3	−8.267	−4.286	3.641	−4.095	5.569

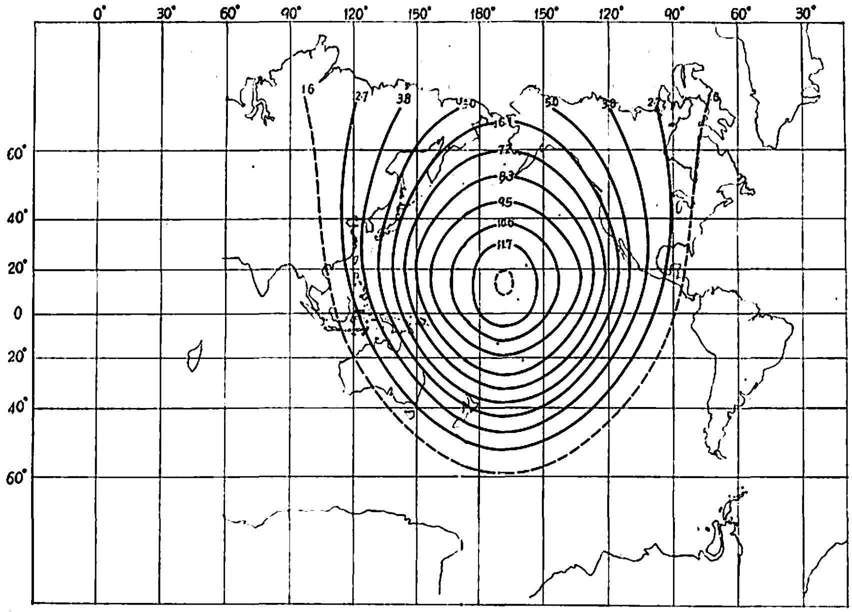

Fig. 1. The distribution of the increment of the total conductivity in the ionosphere. The numerals is expressed in 10^{-7} e. m. u..

where $\Delta\sigma$ is expressed in e.m.u. For any assigned values of k_1^1 and α_1^1, somewhat different values of A_i's will be obtained. But the distribution is not affected greatly. Also, the obtained values of A_i's do not satisfactorily answer the conditions (19) and (20) in the Appendix I. But, the corresponding values to A_n^m's and B_n^m's calculated by means of this coefficients, are consistent with them within an error of 30 %. The map of the geographical distribution is given in Fig. 1. The amounts and/or the region of the increment of the total conductivity are also consistent with those, estimated from the data of the disturbance of the ionosphere, the field strength of VLF, HF radio waves etc. in Japan (20) (21).

Thus we may be able to maintain the idea of the dynamo current in the ionosphere by the natural wind and the increment of the total conductivity due to the nuclear detonation, in this case at least. The amounts of the increment, however, may be overestimated, owing to the assumption that all the quantities of the geomagnetic disturbance, observed by the ordinary magnetograms will be attributed to the dynamo current, without considering the other effects of the detonation such as the field of the magnetically trapped particles and of the hydromagnetic waves. Also the circumstances near the explosion point may be very conspicuous and the validity of this discussion will get more or less small. Moreover, it should be noted that in the assumption of $\Delta\sigma$ are neglected the higher terms of cos $\mathit{\Theta}$ and the obtained distribution of the conductivity is responsible for the one at the time of the nearly maximum disturbance.

§ 3. Conclusions

Assuming that the geomagnetic disturbance observed by the ordinary magnetograms will be perfectly due to the dynamo current in the ionosphere, and also assuming that $\Delta\sigma$ can be expressed in a power series of cos Θ, we estimated $\Delta\sigma$, on the base of the usual dynamo theory of Sq. The result up to the second term of cos Θ is as follows;

$$\Delta\sigma=4.5\times10^{-8}\ (1+2.6\cos\Theta+25.1/\cos^2\Theta)\ \text{in e. m. u..}$$

The expression may be better in principle than the one in the previous paper, by taking into consideration the wind of the velocity potential Ψ_1^1. But, in order to catch the truth, it is desirable to be compared to the world wide data of the ionosphere and also the examination of the temporal changes of the magnetic disturbance will be necessary, as well as the researches of the other effects of the nuclear detonation.

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Appendix I. The calculation of p_n^m

In order to take such discussions as given in the text, the p_n^m in the equation of the spherical harmonic expansion of the current function must be explicitly expressed by the coefficients of the distribution.

For the wind of Ψ_2^2, the functional form obtained by Pratap and Chakrabarty can be directly applied to this case. (22)

For the wind of Ψ_1^1, the obtained results are not at my hand now. Thus we must calculate them.

In place of the declination of the sun δ and the local time t, we take the latitude of the shot point $90^\circ-\theta$ and the difference of the longitude between the observatory and the shot point, respectively. Then we can follow to the dynamo theory of Sq by S. Chapman and/or Pratap et al.

The brief descriptions of calculations are given below (22). We assume that the dynamo current is produced in a spherical shell of mean radius r and thickness e, having only the increment of the conductivity due to the explosion. Making a Fourier resolution of the increment of the conductivity $\Delta\sigma$, we have

$$\Delta\sigma=K\sum_{S'=-\infty}^{\infty} f_s' \cos s' (\varphi-\varphi_0) \quad (\Delta\sigma)^2=K^2\sum_{S=-\infty}^{\infty} g_s \cos s (\varphi-\varphi_0) \tag{1}$$

Then we have

$$\left.\begin{aligned} f_0&=a_0+a_1\beta \cos \theta +a_2\beta^2 \cos^2 \theta +\frac{1}{2}a_2\gamma^2 \sin^2 \theta \\ f_1&=f_{-1}=\frac{1}{2}a_1\gamma \sin \theta+a_2\beta\gamma \sin \theta \cos \theta \\ f_2&=f_{-2}=\frac{1}{4}a_2\gamma^2 \sin^2 \theta \\ f_s'&=f_{-s}'=0 \qquad s'>2 \end{aligned}\right\} \tag{2}$$

and

$$\left.\begin{aligned} g_0=&a_0^2+a_0a_1\beta \cos \theta+(a_1^2+2a_0a_2)[\beta^2 \cos^2 \theta +\frac{1}{2}\gamma^2 \sin^2 \theta] \\ &+2a_1a_2 [\beta^3 \cos^3 \theta+\frac{3}{2}\beta\gamma^2 \cos \theta \sin^2 \theta] \\ &+a_2^2[\beta^4 \cos^4 \theta+\frac{3}{8}\gamma^4 \sin^4 \theta+3\beta^2\gamma^2 \cos^2 \theta\sin^2 \theta] \\ g_1=&g_{-1}=a_0a_1\gamma \sin \theta+(a_1^2+2a_0a_2) \beta\gamma \sin \theta \cos \theta+3a_1a_2 \\ &\times\beta^2\gamma \cos^2 \theta \sin \theta +\frac{3}{4}a_1a_2\gamma^3 \sin^3 \theta \\ &+2a_2^2\beta^3\gamma \cos^3 \theta \sin \theta +\frac{3}{2}a^2{}_2\beta\gamma^3 \cos \theta \sin^3 \theta \\ g_2=&g_{-2}=\frac{1}{4}(a_1^2+2 a_0a_2) \gamma^2 \sin^2 \theta +\frac{3}{2} a_1a_2\beta\gamma^2 \cos \theta \sin^2 \theta \\ &+\frac{3}{2}a^2{}_2\beta^2\gamma^2 \cos^2 \theta \sin^2 \theta+\frac{1}{4}a_2{}^2\gamma^4 \sin^4 \theta \\ g_3=&g_{-3}=\frac{1}{4}a_1a_2\gamma^3 \sin^3 \theta +\frac{1}{2}a^2{}_2\beta\gamma^3 \cos \theta \sin^3 \theta \\ g_4=&g_{-4}=\frac{1}{16}a^2{}_2\gamma^4 \sin^4 \theta \\ g_s=&g_{-s}=0 \qquad s>4 \\ &\text{where } \beta=\cos \theta_0, \quad \gamma=\sin \theta_0 \end{aligned}\right\} \tag{3}$$

We assume that the oscillation of the ionosphere, where the dynamo current is produced, is of the harmonic type having a velocity potential Ψ given by

$$\Psi=\sum_{\tau}\sum_{\sigma} \kappa_\sigma^\tau P_\sigma^\tau \sin [\tau(\varphi-\varphi_0)-\alpha] \tag{4}$$

Then , following the usual dynamo theory, we have

$$a\ (\Delta\sigma)^2\left[\frac{\partial(vH_z)}{\partial\phi}+\frac{\partial(uH_z\sin\theta)}{\partial\theta}\right]=(\Delta\sigma)\left[\frac{1}{\sin\theta}\frac{\partial^2 R}{\partial\phi^2}\right.$$

$$\left.+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial R}{\partial\theta}\right)\right]-\left[\frac{1}{\sin\theta}\frac{\partial R}{\partial\phi}\frac{\partial(\Delta\sigma)}{\partial\phi}+\sin\theta\frac{\partial R}{\partial\theta}\frac{\partial(\Delta\sigma)}{\partial\theta}\right] \quad (5)$$

where R is a current function and H_z is the vertical component of the earth's magnetic field and u, v are the southward and eastward components of the motion of the medium. We take that the earth's field is produced by a central magnetic dipole and approximately that the geomagnetic pole is the geographic pole, then we express

$$H_z=C\cos\theta \quad (6)$$

where C is a constant and is approximately equal to $-\frac{2}{3}$gauss. Also

$$u=\frac{1}{a}\frac{\partial\Psi}{\partial\theta} \qquad v=\frac{1}{a\sin\theta}\frac{\partial\Psi}{\partial\phi} \quad (7)$$

Substituting (1) and (4) into (5), the left-hand side of (5) reduces to

$$K^2\sum_{\sigma}\sum_{\tau}\sum_{S=-\infty}^{\infty} g_s\sin\theta\ \kappa_\sigma^\tau\left[\left\{\frac{\partial H_z}{\partial\theta}\frac{dP_\sigma^\tau}{d\theta}-\sigma(\sigma+1)H_zP_\sigma^\tau\right\}\right.$$
$$\times\sin[\tau(\varphi-\varphi_0)-\alpha]\cos s(\varphi-\varphi_0)+\frac{\tau}{\sin^2\theta}\frac{\partial H_z}{\partial\phi}P_\sigma^\tau$$
$$\cos[\tau(\varphi-\varphi_0)-\alpha]\times\cos s(\varphi-\varphi_0). \quad (8)$$

or

$$K^2\sum_{\sigma}\sum_{\tau}\kappa_\sigma^\tau\sum_{S=-\infty}^{\infty} g_s\sin\theta\left[\left\{\frac{\partial H_z}{\partial\theta}\frac{dP_\sigma^\tau}{d\theta}-\sigma(\sigma+1)H_zP_\sigma^\tau\right\}\right.$$
$$\left.\sin[(\tau+s)(\varphi-\varphi_0)-\alpha]+\frac{\tau}{\sin^2\theta}\frac{\partial H_z}{\partial\phi}P_\sigma^\tau\cos[(\tau+s)(\varphi-\varphi_0)-\alpha]\right]$$

It is clear that R should also be expressed by a surface spherical harmonics and therefore, we can have

$$R=\sum_{\sigma}\sum_{\tau}\kappa_\sigma^\tau CK\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}p_n^mP_n^m\sin[m(\varphi-\varphi_0)-\alpha_m] \quad (9)$$

where p_n^m is a constant to be evaluated.

When m is a negative integer we assume $P_n^m\equiv P_n^{-m}$. Using the relation,

$$\frac{d}{d\theta}\sin\theta\frac{dP_n^m}{d\theta}+\sin\theta\left[n(n+1)-\frac{m^2}{\mathrm{sim}^2\theta}\right]P_n^m=0 \quad (10)$$

the right-hand side of (5) reduces to

$$CK^2 \sum_{\sigma} \sum_{\tau} \sum_{S'=-\infty}^{\infty} \sin\theta \;\; \kappa_\sigma^\tau \sum_n \sum_m p_n^m \Big[\Big\{ \frac{ms'}{\sin^2\theta} - n(n+1) \Big\} f_s' P_n^m - \frac{\partial f_s'}{\partial\theta} \frac{dP_n^m}{d\theta} \Big] \sin\left[(m+s')(\varphi-\varphi_0)-\alpha_m\right] \tag{11}$$

Therefore, equation (5) becomes

$$\sum_{\sigma} \sum_{\tau} \sum_{S=-\infty}^{\infty} \kappa_\sigma^\tau g_s \Big[\Big\{ \frac{\partial H_z}{\partial\theta} \frac{dP_\sigma^\tau}{d\theta} - \sigma(\sigma+1) H_z P_\sigma^\tau \Big\} \sin\left[(\tau+s)(\varphi-\varphi_0)-\alpha\right] + \frac{\tau}{\sin^2\theta} \frac{\partial H_z}{\partial\phi} P_\sigma^\tau \cos\left[(\tau+s)(\varphi-\varphi_0)-\alpha\right] \Big]$$
$$= -C \sum_{\sigma} \sum_{\tau} \kappa_\sigma^\tau \sum_n \sum_m \sum_{S'=-\infty}^{\infty} p_n^m R_n^m(s')\left[(m+s')(\varphi-\varphi_0)-\alpha_m\right] \tag{12}$$

where

$$R_n^m(s') = \Big[n(n+1) - \frac{ms'}{\sin^2\theta} \Big] f_s' P_n^m + \frac{df_s'}{d\theta} \frac{dP_n^m}{d\theta} \tag{13}$$

Thus, we can get p_n^m as functions of the coefficients of the increment of the conductivity. Furthermore, we can calculate p_n^m separately for various sets of values of σ and τ and then sum up them to get the complete expression of R. For any assigned value of σ and τ we have

$$-C \sum_n \sum_m \sum_{S'=-\infty}^{\infty} p_n^m R_n^m(s') \sin\left[(m+s')(\varphi-\varphi_0)-\alpha_m\right]$$
$$= \sum_{S=-\infty}^{\infty} g_s \Big[\Big\{ \frac{\partial H_z}{\partial\theta} \frac{dP_\sigma^\tau}{d\theta} - \sigma(\sigma+1) H_z P_\sigma^\tau \Big\} \sin\left[(\tau+s)(\varphi-\varphi_0)-\alpha\right]$$
$$+ \frac{\tau}{\sin^2\theta} \frac{\partial H_z}{\partial\phi} P_\sigma^\tau \cos\left[(\tau+s)(\varphi-\varphi_0)-\alpha\right] \tag{14}$$

Hence, taking $H_z = C\cos\theta$ we have

$$\sum_n \sum_m \sum_{S'=-\infty}^{\infty} p_n^m R_n^m(s') \sin\left[(m+s')(\varphi-\varphi_0)-\alpha_m\right]$$
$$= \frac{1}{2\sigma+1} \{ \sigma(\sigma+2)(\sigma-\tau+1) P_{\sigma+1}^\tau + (\sigma^2-1)(\sigma+\tau) P_{\sigma-1}^\tau \}$$
$$\times \sum_{S=-\infty}^{\infty} g_s \sin\left[(\tau+s)(\varphi-\varphi_0)-\alpha\right] \tag{15}$$

Comparing the coefficients of the corresponding harmonic terms, we get $\alpha_m = \alpha$ for all values of m and

$$\sum_n\sum_m p_n^m R_n^m(s') = \frac{1}{2\sigma+1}\{\sigma(\sigma+2)(\sigma-\tau+1)P_{\sigma+1}^{\tau}$$

$$+(\sigma^2-1)(\sigma+\tau)P_{\sigma-1}^{\tau}\}g_s \tag{16}$$

where

$$s' = s+\tau-m$$

If we assume the conductivity is expressed as given in the text, we have

$$\begin{aligned} g_s = g_{-s} &= 0 \qquad s>4 \\ R_n^m(s') &= 0 \qquad s'>2 \end{aligned} \tag{17}$$

By the way, we take the wind of Ψ_1^1 and Ψ_2^2 in the text. And then we will calculate the coefficient $p_n{}^m$ for the cases $\sigma=\tau=1$ and $\sigma=\tau=2$.

Case 1. $\sigma=\tau=1$

In this case, equation (16) reduces to

$$g_s P_2{}^1 = \Sigma\Sigma p_n^{s+1-s'} R_n^{s+1-s'} \tag{18}$$

When we use the expression for g_s given by (3) and compare the corresponding terms of the both sides of (18), then we get the values of $p_n{}^m$. In this way, we can solve exactly equation (18) for $s \geqq 2$ and $s=-1$. But we must assume the following to satisfy (18) for $s=1$, 0, -2, -3, and -4.

$$4\,a_0 - 6\frac{a_2}{a_0} + \frac{695}{1008}\frac{a_2}{a_0} = 0 \tag{19}$$

If the above equation is satisfied, equation (18) can be solved exactly for $s \geqq -1$, although the small residuals are left behind for $s=-2$, -3, -4. The residuals, however, are only a small part percentages of the complete expressions that occur on the right side of the corresponding equations. And also, most of them vanish when $\beta=0$. The results are given in Table I.

Case 2. $\sigma=\tau=2$

For this values of σ and τ, Chakrabarty and Pratap made a tedious calculation in a similar manner and gave the results in their paper (22). Then, we only reproduce here them for the convenience in Table II. In order to solve equation exactly, they assumed

$$\frac{2}{3}a_0 a_2 - \frac{1}{4}a_1{}^2 = 0 \tag{20}$$

Table I. p_n^m for ψ_1^1 $(\sigma=\tau=1)$

p_1^0	$\frac{219}{60}\sqrt{3}\,a_1\gamma-\frac{1351}{2016}\sqrt{3}\frac{a_1a_2}{a_0}\gamma_2+\frac{695}{1008}\sqrt{3}\,a_2\beta^2$
p_2^0	$-3\sqrt{3}\frac{a_1^2\beta}{a_2\gamma}+\frac{2}{56}\sqrt{3}\,a_2\beta\gamma$
p_3^0	$-\frac{2}{5}\sqrt{3}\,a_1\frac{\beta^2}{\gamma}-\frac{13}{45}\sqrt{3}\,a_1$
p_4^0	$-\frac{2}{105}\sqrt{3}\,a_2\beta\gamma$
p_1^1	$-\frac{152}{15}\sqrt{6}\,a_1\beta-\frac{1}{4}\sqrt{6}\frac{a_0a_2\beta}{a_1}$
p_2^1	$\frac{1}{6}a_0+3\frac{a_1^2}{a_2}+\frac{1}{28}a_2\beta^2-\frac{1}{56}a_2\gamma^2$
p_3^1	$\frac{26}{45}\sqrt{2}\,a_1\beta$
p_4^1	$\frac{\sqrt{30}}{210}a_2\beta^2-\frac{\sqrt{30}}{420}a_2\gamma^2$
p_2^2	$\frac{9}{84}a_2\beta\gamma$
p_3^2	$-\frac{\sqrt{5}}{9}a_1\gamma$
p_4^2	$\frac{\sqrt{15}}{105}a_2\beta\gamma$
p_3^3	0
p_4^3	$\frac{\sqrt{210}}{840}a_2\gamma^2$
p_4^4	0
p_1^{-1}	$-\frac{470}{9}a_2\beta^2-\frac{87}{56}a_2\beta\gamma^2-\frac{272}{3}a_0\frac{\beta^2}{\gamma^2}+68\frac{a_1^2\beta^2}{a_2\gamma^2}$ $+18\frac{a_1^2}{a^2}\beta-\frac{136}{3}a_2\frac{\beta^4}{\gamma^2}$
p_2^{-1}	$-4a_0-3\frac{a_1^2}{a_2}+\frac{1405}{1008}a_2\gamma$ $-\frac{68}{3}a_0\frac{\beta^2}{\gamma^2}-\frac{34}{3}a_2\frac{\beta^4}{\gamma^2}$
p_3^{-1}	$-2\sqrt{2}\,a_1\beta-\frac{64}{15}\sqrt{2}\,a_1\frac{\beta^3}{\gamma^2}$
p_4^{-1}	$-\frac{\sqrt{30}}{840}a_2\gamma^2$
p_2^{-2}	$\frac{136}{3}a_0\frac{\beta}{\gamma}-\frac{106}{9}a_2\frac{\beta^3}{\gamma}$ $+\frac{194}{9}a_2\beta\gamma$
p_3^{-2}	$-\frac{2}{5}\sqrt{5}\,a_1\gamma+\frac{116}{45}\sqrt{5}\,a_1\frac{\beta^2}{\gamma}$
p_4^{-2}	0
p_3^{-3}	$-\frac{68}{45}\sqrt{30}\,a_1\beta$
p_4^{-3}	0
p_4^{-4}	0

Table II. p_n^m for ψ_2^2 $(\sigma=\tau=2)$*

p_1^0	$8a_0-\frac{56}{5}a_0\frac{1}{\gamma^2}+\frac{48}{5}a_2\frac{\beta^2}{\gamma^2}+\frac{36}{35}a_2\gamma^2$	p_5^3	$\frac{4}{1575}a_2\beta\gamma$
p_2^0	$-\frac{4}{5}a_1\frac{\beta}{\gamma^2}$	p_4^4	0
p_3^0	$-\frac{4}{15}a_2\gamma^2$	p_5^4	$\frac{1}{3150}a_2\gamma^2$
p_4^0	0	p_1^{-1}	$\frac{8a_0^2\beta}{a_2\gamma^3}-\frac{112}{5}a_0\frac{\beta}{\gamma^3}+20a_0\frac{\beta}{\gamma}+\frac{96}{5}a_2\frac{\beta^3}{\gamma^3}-\frac{24}{5}a_2\frac{\beta^3}{\gamma}$
p_5^0	$\frac{4}{105}a_2\gamma^2$		
p_1^1	$(2a_0-\frac{12}{5}a_2)\frac{\beta}{\gamma}+\frac{72}{35}a_2\beta\gamma$	p_2^{-1}	$\frac{24}{5}a_1\frac{1}{\gamma^3}-\frac{18}{5}a_1\frac{1}{\gamma}$
p_2^1	$\frac{1}{7}a_1\gamma+\frac{1}{5}a_1\frac{1}{\gamma}$	p_3^{-1}	0
p_3^1	$\frac{2}{15}a_2\beta\gamma$	p_4^{-1}	0
p_4^1	$-\frac{3}{70}a_1\gamma$	p_5^{-1}	0
p_5^1	$-\frac{16}{525}a_2\beta\gamma$	p_2^{-2}	$-\frac{96}{5}a_1\frac{\beta}{\gamma^4}(1+\beta^2)$
p_2^2	$\frac{1}{14}a_1\beta$	p_3^{-2} ~ p_5^{-4}	0
p_3^2	$\frac{2}{15}a_0+\frac{2}{75}a_2$		
p_4^2	$\frac{1}{35}a_1\beta$		
p_5^2	$\frac{4}{525}a_2(\beta^2-\frac{1}{2}\gamma^2)$		
p_3^3	$\frac{1}{45}a_2\beta\gamma$		
p_4^3	$\frac{1}{140}a_1\gamma$		

* This table is calculated for the associated Legendre's functions.

Appendix. II. Geomagnetic Effects on Oct. 22 and 28, 1962.

When we prepared this note, we received a letter from Dr. S. L. Goldblatt, Director of Geophysics, NRA. He requested the data of the nuclear explosion on Oct. 22, and 28, 1962. Considering the general availability, we reproduced some of the

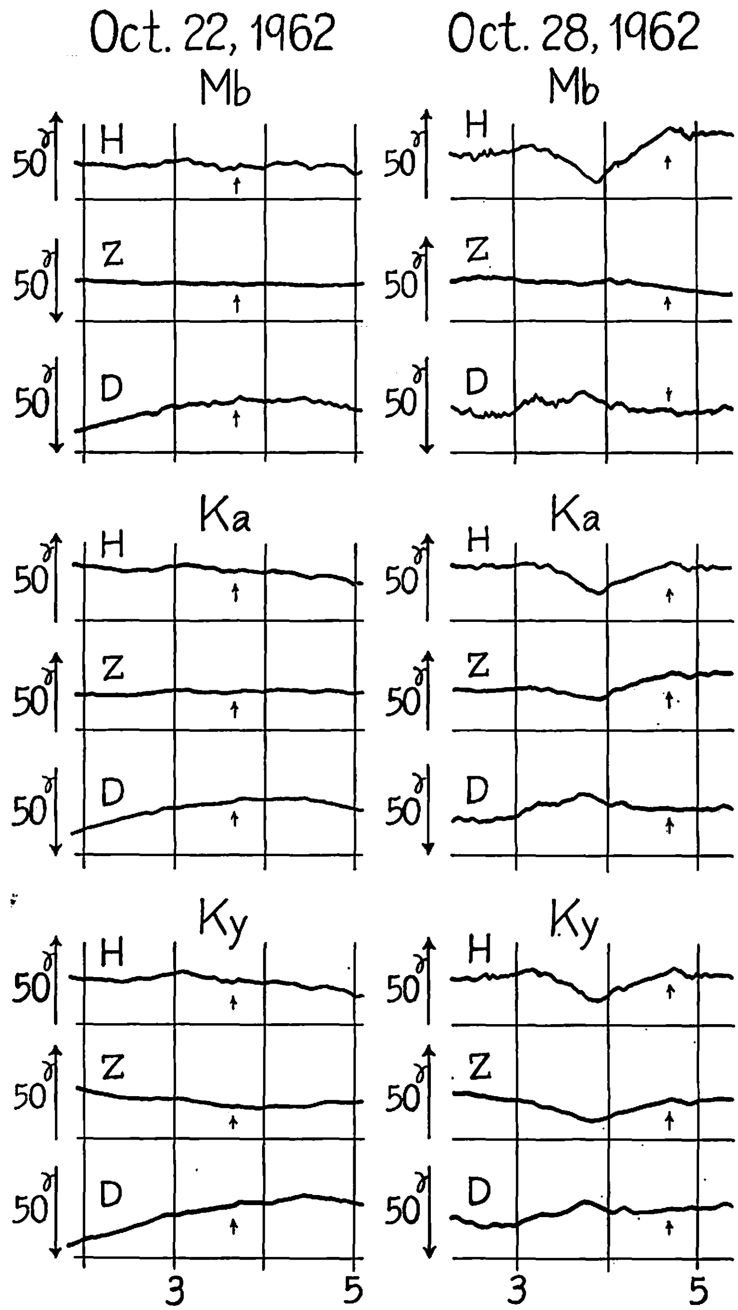

Fig. 2. The ordinary magnetograms of Memambetsu, Kakioka and Kanoya around the times of the explosions on Oct. 22, 1962 (Left) and Oct. 28, 1962 (Right). The arrows on the margin indicate the directions of increasing, and the scale values for each component.

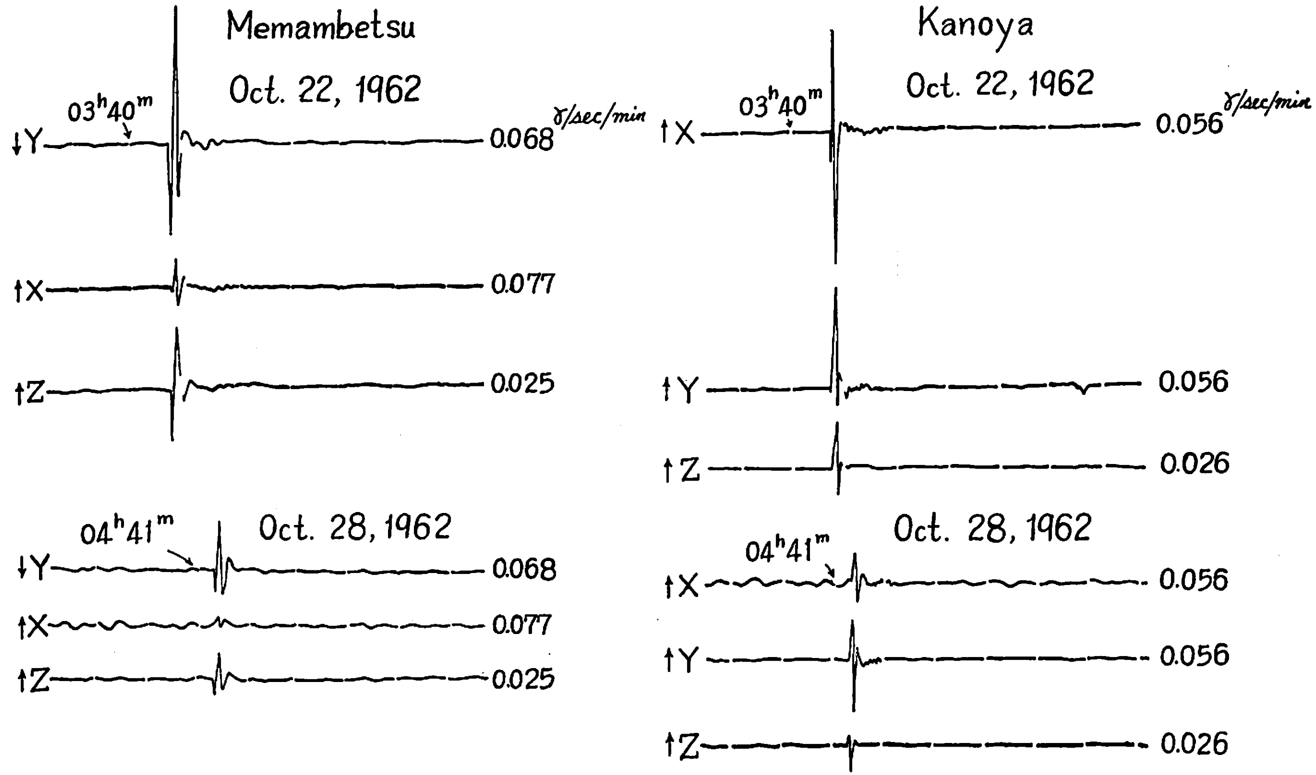

Fig. 3. The induction magnetograms (loop) of Memambetsu and Kanoya around the times of the nuclear explosions on Oct. 22, 1962 and Oct. 28, 1962. The arrows on the margin indicate the directions of increaasing and the numerals are the scale values on the original sheets, where one minute (between the adjacent breaks) is expressed by the length of 12 mm.

geomagnetic data at our observatory. In these cases, the disturbances on the ordinary magnetograms are too small to be examined in such a way as one in the text, but the disturbances observed by the induction magnetograms may be rather sufficiently simple for the researches of the hydromagnetic wave effect.

Table III. Geographic and Geomagnetic Coordinates of Observatories.

Observatory	Geographic Lat.	Geographic Long.	Geomagnetic Lat.	Geomagnetic Long.
Memambetsu	43°55′N	144°12′E	34.0°	208.4°
Kakioka	36°14′N	140°11′E	26.0°	206.0°
Kanoya	31°25′N	130°53′E	20.5°	198.1°

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超高空核爆発による地磁気変化—補遺

山　口　又　新

概　要

核爆発による地磁気擾乱は，（1）電磁流体波（2）磁力線に捕捉された粒子の運動による影響（3）電離層内を流れる電流による擾乱或はその二又は三者の重畳と考えられるが，前報文（地磁気観測所要報第11巻第1号）では，1962年7月9日にヂョンストン島で行われた超高空核実験による地磁気異常変化について報告し，地磁気の通常記録に現われた変化を，第三の作用によるものを仮定し，dynamo 理論から核爆発の何らかの作用によつて生ずるであらうと思われる電離層内の電気伝導度異常増加を評価した。この場合の風は自然のもので第二調和波のみをとり扱つたが，本報文では，第一，第二調和波を同時に考慮した。尚最後に，1962年10月22日および28日の核実験時の地磁気記録を付してある。